Feb 21

tame Frechet space

<u>Def</u> A tame Frechet space F is a vector space with a grading semi-norm

$$\|\cdot\|_n : F \longrightarrow \mathbb{R}^+, \quad n \in \mathbb{N}$$

i.e. $\|f\|_0 \leq \|f\|_1 \leq \cdots \leq \|f\|_n \quad \forall f \in F$.

$$\longrightarrow \quad d(f, g) = \sum_{n=0}^{\infty} \frac{1}{2^n} \cdot \frac{\|f-g\|_n}{1+\|f-g\|_n}$$

(F, d) is complete, satisfies the <u>tame condition</u>.

<u>Def</u> Tame maps

$P : U \subseteq F \longrightarrow G$ satisfies a tame estimate of <u>degree</u> r and <u>base</u> b if

$$\|P(f)\|_n \leq C_n \|f\|_{n+r} \quad \forall f \in U, \ \forall n \geq b$$

(Banach spaces are Frechet spaces, semi norm does not satisfies positive definiteness)

$\exists B$ Banach space (P)

$$\begin{tikzcd} F \arrow[rr, "id"] \arrow[dr, "L"'] & \arrow[d, phantom, "\circlearrowright"] & F \\ & \Sigma(B) \arrow[ur, "M"'] & \end{tikzcd}$$

L, M are linear tame maps.

$$\Sigma(B) = \{ (x_n) \in B \mid \sup_k e^{nk} \|x_k\|_B < \infty \}$$

$$\|(x_k)\|_n = \sup_k (e^{nk} \|x_k\|_B)$$

$B \rightsquigarrow \Sigma(B)$

Banach Frechet

F satisfies (P) is a tame Frechet space

Ex. $C^\infty(X, E)$, E a vector bundle

$\|\cdot\|_{C^k}$ $\|\cdot\|_{C^{k,\alpha}}$ $\|\cdot\|_{W^{k,p}}$

Nash-Moser Thm (see last time)

Hamilton's existence thm

E. nonlinear deg 2.

L. integrability cond of E.

$L(f)(g)$ is deg 1 on both f and g.

Consider $\mathcal{E} : C^\infty(X \times [0,1], F) \longrightarrow C^\infty(X \times [0,1], F) \times C^\infty(X, F)$

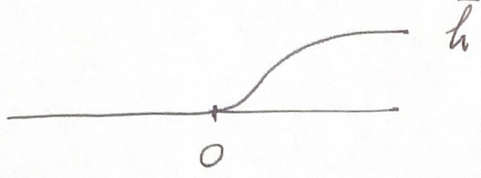
$$f \longmapsto \left(\frac{\partial f}{\partial t} - E(f),\ f|_{t=0}\right)$$

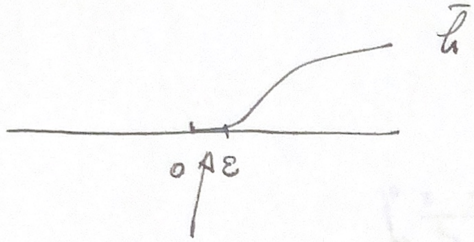
Assume local invertibility of $\mathcal{E}$ holds.

Let f has a Taylor expansion in t formally solves

$$\begin{cases} \dfrac{\partial f}{\partial t} = E(f_0) \\ f|_{t=0} = f_0 \end{cases}$$

Let $\bar{h} = \frac{\partial \bar{f}}{\partial t} - E(\bar{f}) = O(t^{\infty})$





translation

$\bar{h} = 0 \quad 0 \leq t \leq \varepsilon$